Second Order Nonuniform Grid Spacing in the FDTD Technique

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ABSTRACT:

A second order accurate nonuniform technique for FDTD simulations which does not depend on supraconvergence is presented. The technique is useful for FDTD simulations with nonuniform grid spacings for systems which require finer details in small regions of the simulation space.

FORMULATION:

The FDTD method has been widely used to solve complex electromagnetic problems in three dimensions including transmission lines, antennas, IC packages, etc. One of the limitations of the method has been its dependence on uniform grid spacing which leads to large computer memory requirements when modeling small details inside the simulation space, such as the modeling of submicron on chip interconnects[1] . The method is second order accurate with respect to the grid increment when implemented on a uniform grid. Supraconvergence has been used to show that if the grid is made nonuniform so that the electric field differencing is first order, and the magnetic and time differencings remain second order, then the whole system should behave as a second order accurate system [2]. However, this approach has been shown to be sensitive to the accuracy of the boundary conditions associated with the boundary of the simulation space [3]. If the boundary conditions are not highly accurate, the system will not exhibit second order accuracy. Additionally there will still be local first order errors which may influence measurements based on transient data.

In this paper a second order FDTD method is implemented using an algorithm which generates the coefficients for the nonuniform portions of the grid and achieves second order accuracy by utilizing three neighboring fields instead of the standard two neighboring fields used in centered differencing [4]. The nonuniform coefficients are derived using the method of undetermined coefficients, shown to be second order accurate using a Taylor's series expansion, and the stability criteria is derived using Fourier analysis. This new nonuniform technique allows second order accuracy that is not dependent on the boundary conditions and does not have local first order errors.

The stability criteria for three dimensional simulations was rigorously derived and is found to be (Equation 1). This stability criteria can be shown to be less restrictive than the standard stability criteria based on the smallest space step. Since the grid is orthogonal, the nonuniform second order technique requires virtually no additional memory and requires up to six additional multiplications and additions per nonuniform cell. The standard lossy FDTD can be implemented using 36 multiplies and 24 additions per cell. There was no noticeable speed difference between the first and second order nonuniform techniques.

NUMERICAL EXPERIMENT:

An enclosed stripline, Figure 1, is used to test the second order technique compared to the first order technique. The comparison is of the FDTD measured impedance of the

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transmission line as a function of frequency for various grid discontinuities. The impedance of the enclosed stripline is calculated one cell away from the discontinuity in the grid spacing using the procedure described by Taflove [5].

The structure is excited by a Gaussian pulse in a cross sectional plane perpendicular to the direction of propagation. The field pattern in the input plane is determined by a 2-D Laplace solver which insures that the fields in the enclosed stripline are essentially TEM. The input plane was located near the center of the enclosed stripline structure and the simulation was terminated before reflections from the end of the stripline could interact with the measurements.

For all of the experiments described in this paper, $\Delta x=\Delta y=\Delta z=10[\mu m]$ and the time step was set to 0.45 of the timestep determined by the standard FDTD stability criteria. All metal structures are modeled as ideal conductors and infinitely thin.

RESULTS:

Figure 2 shows the results of the stripline impedance measurement using uniform grid spacing. Note that the frequency axis is stopped at 3[THz] which corresponds to a wavelength that is ten grid spaces in length [5,6]. In the nonuniform plots, the vertical line corresponds to the frequency at which the largest grid spacing is equal to a tenth of a wavelength.

Figure 3 and 4 compare the results for an abrupt change in the grid spacing from Δz to $2*\Delta z$. The performance of the second order nonuniform differencing is much better than the first order results in the region where wavelength is greater than ten grid spacings.

Figures 5 and 6 show the results for a decreasing grid size of Δz to $1/2*\Delta z$. Again as expected the second order nonuniform technique performs better than the first order technique.

Figure 7 shows a composite of first and second order results for $1.5*\Delta z$, $2.0*\Delta z$, and $3.0*\Delta z$. The frequencies for a tenth of a wavelength are 2[THz], 1.5[THz], and 1[THz] respectively.

CONCLUSIONS

A second order accurate technique for nonuniform FDTD has been implemented and compared to the current first order techniques which rely on supraconvergence. It is seen on nonuniform grids that the new second order technique produces better results for a broader frequency range than the first order techniques. The same approach for generating the nonuniform space stepping can also be applied to the time stepping. Although nonuniform time stepping would increase the memory requirements, more past time information needs to be stored, it would allow for the FDTD method to be coupled with nonlinear simulators which rely on time stepping. The nonuniform second order technique should also work within nonuniform PML boundaries and may lead to more efficient implementations.

ACKNOWLEDGMENT

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$$\Delta t \le \frac{1}{c\sqrt{\left(\frac{2\Delta x_3 + 6\Delta x_1}{\Delta x_2\left(\Delta x_1 + \Delta x_2\right)\left(\Delta x_3 + 2\Delta x_1 + \Delta x_2\right)}\right) + \left(\frac{2\Delta y_3 + 6\Delta y_1}{\Delta y_2\left(\Delta y_1 + \Delta y_2\right)\left(\Delta y_3 + 2\Delta y_1 + \Delta y_2\right)}\right) + \left(\frac{2\Delta z_3 + 6\Delta z_1}{\Delta z_2\left(\Delta z_1 + \Delta z_2\right)\left(\Delta z_3 + 2\Delta z_1 + \Delta z_2\right)}\right)}}$$

Equation (1) Nonuniform Stability Criteria

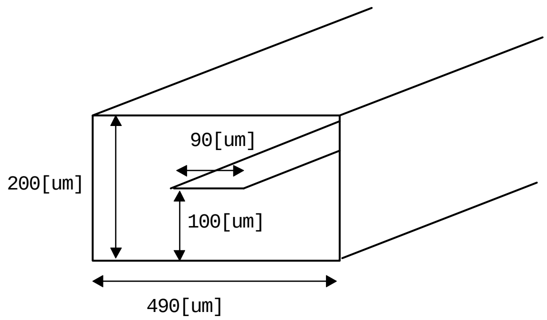

Figure 1 Enclosed Stripline Structure

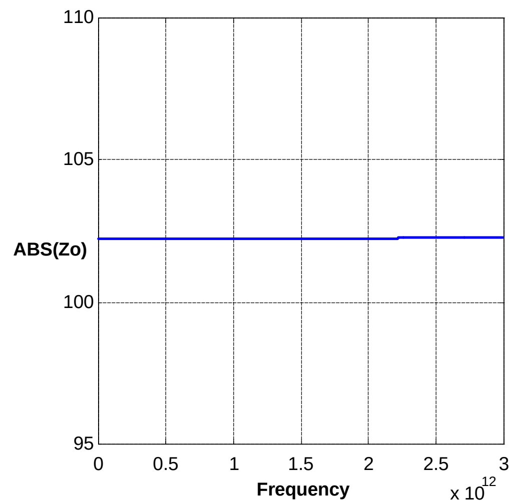

Figure 2 Uniform Grid Results

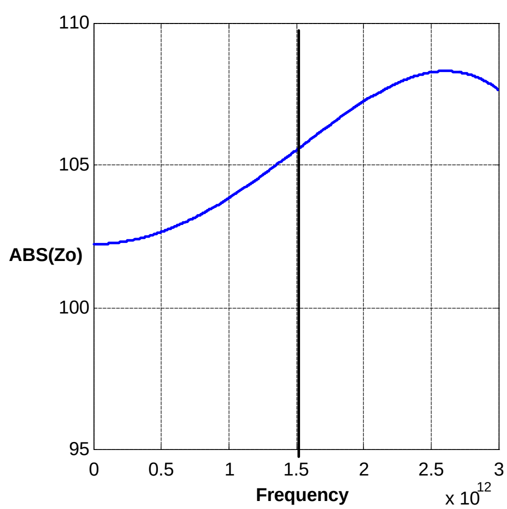

Figure 3 First Order Δz=>2.0 Δz

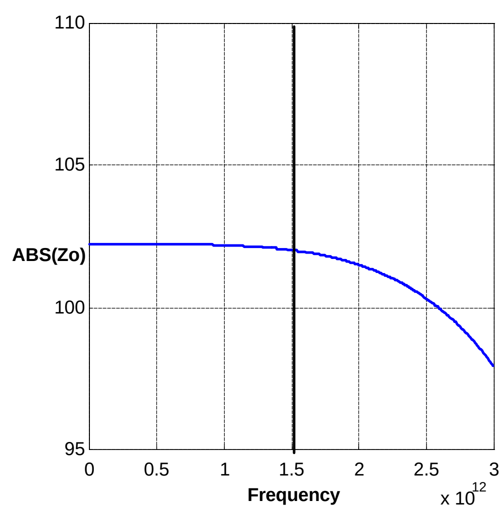

Figure 4 Second Order Δz=>2.0 Δz

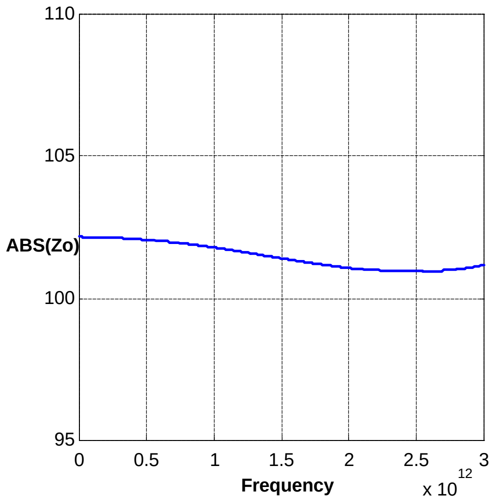

Figure 5 First Order Δz=>0.5Δz

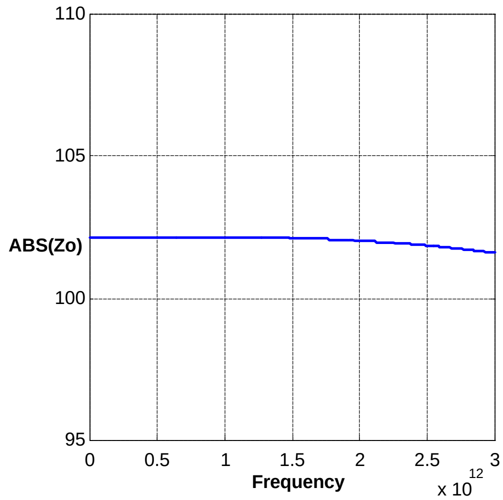

Figure 6 Second Order Δz=>0.5Δz

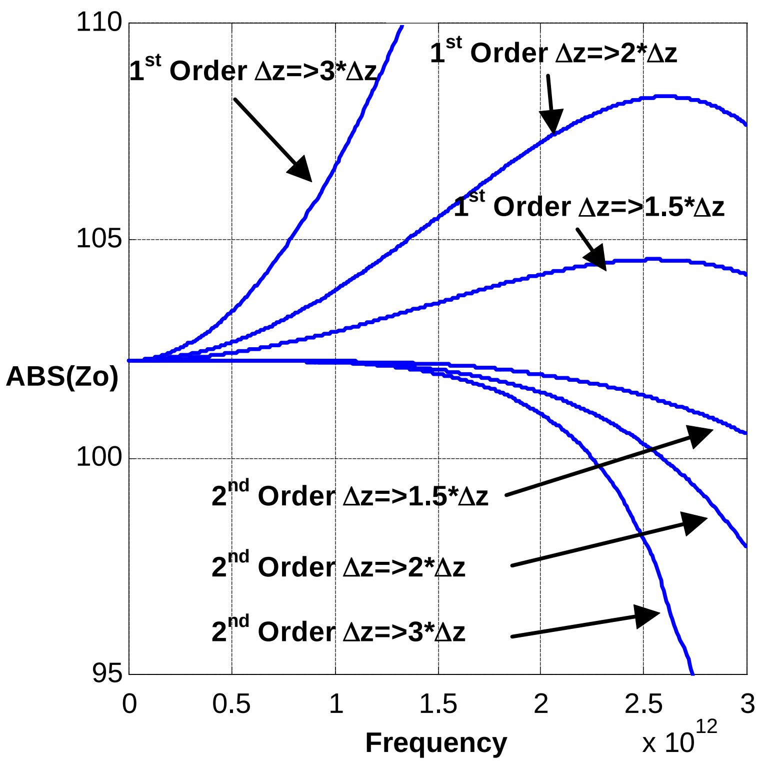

Figure 7 Composite of First and Second Order Results